Whether conformal supersymmetry is broken by quantum p-branes with exotic supersymmetry?

D.V. Uvarov[a] and A.A. Zheltukhin[a,b]

[a] Kharkov Institute of Physics and Technology, 61108 Kharkov, Ukraine

[b] Institute of Theoretical Physics, University of Stockholm, Albanova, SE-10691 Stockholm, Sweden

Abstract

Classical and quantum symmetries of super p-branes preserving exotic $\frac{3}{4}$ fraction of $N = 1 D = 4$ global supersymmetry are studied. Classical realization of the algebra of global and world-volume symmetries is constructed and its quantum generalizations are analyzed. Established is that the status of the conformal supersymmetry $OSp(1|8)$ as a proper quantum symmetry of brane depends both on the choice of its vacuum state and the associated ordering of $\hat{\mathcal{Q}}$ and $\hat{\mathcal{P}}$ operators.

1 Introduction

Studying the symmetries of M-theory is one of the sharp problems and it is under wide investigations which reveal new connections [1], [2], [3] between space-time symmetries and BPS vacuum states permitted by the extensions of the Poincare superalgebra by the tensorial central charges (TCC) $Z_{m_1 \ldots m_p}$ [4], [5]. Using TCC and coordinates associated with them has opened new views for the construction of supersymmetric generalizations of field and string/brane models [6]-[18]. An interesting effect of the TCC inclusion is a possibility to regulate the fraction of spontaneously broken supersymmetry in the models based on the extended superalgebras [19]-[26]. On the other hand, the (super)space-times enlarged by the TCC coordinate addition provide well known framework for correct formulation of higher spin field theory [27], [28] and its connection with the conformal $\mathcal{N} = 4$ super Yang-Mills theory [29], [30]. These promising results stimulate interest in studying local and global quantum symmetries of extended objects propagating in the superspaces enlarged by TCC coordinates.

A new exactly solvable twistor-like model of (super)string and super p-brane linear in the derivatives and preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of $N = 1$ supersymmetry in the enlarged superspace, similarly to the superparticle model [19], was studied in [31],[32]. Because of the $OSp(1|2\mathsf{M})$ global symmetry of the model, its static p-brane solution was formulated in terms of symplectic supertwistors previously used while studying superparticle models [33], [34], [19] and forming a subspace of the $Sp(2\mathsf{M})$ invariant symplectic space [27], [28]. For the particular case of $D = 11$ the model [31] is invariant under the $OSp(1|64)$ symmetry and it was analyzed in [35] and further generalized in [36]. The general static solutions of the model [31] describe string/brane configurations which spontaneously break $OSp(1|2\mathsf{M})$ global conformal supersymmetry. The partially spontaneously broken character of the $OSp(1|2\mathsf{M})$ symmetry

was also observed in the Wess-Zumino like super p-brane models [37] nonlinear in derivatives and preserving $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of supersymmetry. The models [37] generate the Dirichlet boundary conditions for the superstrings and superbranes as a consequence of the variational principle. These results unified with the observation concerning nonlinear realization of $OSp(1|64)$ supersymmetry in $D = 11$ supergravity [38] support the tempting conjecture [28] that string/M-theory is a spontaneously broken phase of higher spin gauge theory.

The present paper continues the investigation of the Hamiltonian and quantum structure of the model of tensionless super $p-$branes [31] started in [39]. This model is characterized by the primary and secondary constraints which form a mixture of the first- and the second-class ones. These constraints were covariantly split in [39], where the corresponding Dirac brackets were constructed and the D.B. and operator realizations of the orthosymplectic algebra in $D = 4\, N = 1$ enlarged superspace were obtained.

Here we consider an alternative approach to studying the Hamiltonian structure of the model which is based on the conversion procedure [40]-[42]. The application of this procedure gives a possibility to transform all the primary constraints into the first-class constraints that efficiently simplifies investigations of the global and gauge symmetries of p-brane on the quantum level. As a result, we obtain the properly modified expressions for the generator densities of $OSp(1|8)$ superalgebra on the classical level ensuring validity of the global $OSp(1|8)$ supersymmetry in the phase space extended by conversion variables. Then we consider operator realizations of the constraints, identified with the generators of world-volume gauge symmetry, and generators of the generalized conformal supersymmetry. The question whether the classical global symmetry $OSp(1|8)$ of the exotic superstring and superbranes is explicitly broken by quantization is investigated.

We show that the status of $OSp(1|8)$ conformal supersymmetry as a proper quantum symmetry of exotic BPS brane depends both on the choice of its vacuum state and the corresponding physical ordering of the coordinate and momentum operators.

2 Tensionless super p-branes with TCC coordinates: Lagrangian formulation and primary constraints

A simple model describing tensionless strings and $p-$branes evolving in the symplectic superspace $\mathcal{M}^{susy}_{\mathsf{M}}$ was recently proposed in [31]. For $\mathsf{M} = 2^{[\frac{D}{2}]}$ ($D = 2,3,4\ mod 8$) the superspace $\mathcal{M}^{susy}_{\mathsf{M}}$ includes standard D-dimensional Minkowski space-time coordinates $x_{ab} = x^m(\gamma_m C^{-1})_{ab}$ accompanied by the Majorana spinor θ_a ($a = 1, 2, ..., 2^{[\frac{D}{2}]}$) and the tensor central charge coordinates z_{ab} additively unified with the space-time coordinates in the symmetric spin-tensor Y_{ab}. The action of the model [31]

$$S_p = \frac{1}{2}\int d\tau d^p\sigma\ \rho^\mu U^a W_{\mu ab} U^b \tag{1}$$

includes the world-volume pullback

$$W_{\mu ab} = \partial_\mu Y_{ab} - 2i(\partial_\mu\theta_a\theta_b + \partial_\mu\theta_b\theta_a) \tag{2}$$

of the supersymmetric Cartan differential one-form $W_{ab} = W_{\mu ab}d\xi^\mu$, where $\partial_\mu \equiv \frac{\partial}{\partial\xi^\mu}$ and $\xi^\mu = (\tau, \sigma^M)$, $M = 1, 2, ..., p$ are world-volume coordinates. The local auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ parametrizes the generalized momentum $P^{ab} = \rho^\tau U^a U^b$ of the tensionless

$p-$brane and $\rho^\mu(\tau, \sigma^M)$ is the world-volume vector density providing the reparametrization invariance of S_p. The supersymmetric and world-volume reparametrization invariant action (1) has $(\mathsf{M}-1)$ $\kappa-$symmetries

$$\delta_\kappa\theta_a = \kappa_a, \quad \delta_\kappa Y_{ab} = -2i(\theta_a\kappa_b + \theta_b\kappa_a), \quad \delta_\kappa U^a = 0, \tag{3}$$

where the transformation parameter $\kappa_a(\xi)$ is restricted by only one real condition

$$U^a\kappa_a = 0, \tag{4}$$

as it follows from the transformation rules of the primary variables θ_a, U_a and $Y_{ab} = x_{ab} + z_{ab}$. As a result, the action (1) preserves $\frac{\mathsf{M}-1}{\mathsf{M}}$ fraction of the original $N=1$ global supersymmetry.

$D=4$ $N=1$ supersymmetry algebra takes the form

$$\{Q_a, Q_b\} = (\gamma^m C^{-1})_{ab}P_m + i(\gamma^{mn}C^{-1})_{ab}Z_{mn} \tag{5}$$

including the TCC two-form Z_{mn} with the charge conjugation matrix C chosen to be imaginary in the Majorana representation. The string/brane model (1) is formulated in generalized $(4+6)$-dimensional space $\mathcal{M}_{4+6}$ extended by the Grassmannian Majorana bispinor θ_a. In the Weyl basis real symmetric 4×4 matrix Y_{ab} acquires the form

$$\begin{gathered}Y_{ab} = x_{ab} + z_{ab} = x_m(\gamma^m C^{-1})_{ab} + z_{mn}(\gamma^{mn}C^{-1})_{ab}, \\ Y_a{}^b = Y_{ad}C^{db} = \begin{pmatrix} z_\alpha{}^\beta & x_{\alpha\dot\beta} \\ \tilde{x}^{\dot\alpha\beta} & \bar{z}^{\dot\alpha}{}_{\dot\beta} \end{pmatrix},\end{gathered} \tag{6}$$

and the auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ is presented as

$$U_a = \begin{pmatrix} u_\alpha \\ \bar{u}^{\dot\alpha} \end{pmatrix} \tag{7}$$

with the charge conjugation matrix C

$$C^{ab} = \begin{pmatrix} \epsilon^{\alpha\beta} & 0 \\ 0 & \epsilon_{\dot\alpha\dot\beta} \end{pmatrix}. \tag{8}$$

Then the action (1) acquires the form

$$S_p = \frac{1}{2}\int d\tau d^p\sigma\ \rho^\mu \left(2u^\alpha\omega_{\mu\alpha\dot\alpha}\bar{u}^{\dot\alpha} + u^\alpha\omega_{\mu\alpha\beta}u^\beta + \bar{u}^{\dot\alpha}\bar\omega_{\mu\dot\alpha\dot\beta}\bar{u}^{\dot\beta}\right), \tag{9}$$

where the supersymmetric one-forms $\omega_{\mu\alpha\dot\alpha}$ and $\omega_{\mu\alpha\beta}$ are

$$\begin{aligned}\omega_{\mu\alpha\dot\alpha} &= \partial_\mu x_{\alpha\dot\alpha} + 2i(\partial_\mu\theta_\alpha\bar\theta_{\dot\alpha} + \partial_\mu\bar\theta_{\dot\alpha}\theta_\alpha), \\ \omega_{\mu\alpha\beta} &= -\partial_\mu z_{\alpha\beta} - 2i(\partial_\mu\theta_\alpha\theta_\beta + \partial_\mu\theta_\beta\theta_\alpha), \\ \bar\omega_{\mu\dot\alpha\dot\beta} &= -\partial_\mu\bar{z}_{\dot\alpha\dot\beta} - 2i(\partial_\mu\bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} + \partial_\mu\bar\theta_{\dot\beta}\bar\theta_{\dot\alpha}).\end{aligned} \tag{10}$$

Following [39] let us introduce the momenta densities $\mathcal{P}^{\mathfrak{M}}(\tau, \sigma^M)$

$$\mathcal{P}^{\mathfrak{M}} = \frac{\partial L}{\partial\dot{\mathcal{Q}}_{\mathfrak{M}}} = (P^{\dot\alpha\alpha}, \pi^{\alpha\beta}, \bar\pi^{\dot\alpha\dot\beta}, \pi^\alpha, \bar\pi^{\dot\alpha}, P_u^\alpha, \bar{P}_u^{\dot\alpha}, P_\mu^{(\rho)}) \tag{11}$$

canonically conjugate to the coordinates $\mathcal{Q}_{\mathfrak{M}} = (x_{\alpha\dot\alpha}, z_{\alpha\beta}, \bar z_{\dot\alpha\dot\beta}, u_\alpha, \bar u_{\dot\alpha}, \rho^\mu)$ with respect to the Poisson brackets

$$\{\mathcal{P}^{\mathfrak{M}}(\vec\sigma), \mathcal{Q}_{\mathfrak{N}}(\vec\sigma')\}_{P.B.} = \delta^{\mathfrak{M}}_{\mathfrak{N}} \delta^p(\vec\sigma - \vec\sigma') \tag{12}$$

with the periodic δ−function $\delta^p(\vec\sigma - \vec\sigma')$, where $\vec\sigma = (\sigma^1, ..., \sigma^p)$, for the case of closed string/brane studied here. The explicit form of P.B. (12) is given by the expressions

$$\begin{gathered}\{P^{\dot\alpha\alpha}(\vec\sigma), x_{\beta\dot\beta}(\vec\sigma')\}_{P.B.} = \delta^\alpha_\beta \delta^{\dot\alpha}_{\dot\beta} \delta^p(\vec\sigma - \vec\sigma');\\ \{\pi^{\alpha\beta}(\vec\sigma), z_{\gamma\delta}(\vec\sigma')\}_{P.B.} = \tfrac{1}{2}(\delta^\alpha_\gamma \delta^\beta_\delta + \delta^\alpha_\delta \delta^\beta_\gamma)\delta^p(\vec\sigma - \vec\sigma');\\ \{\bar\pi^{\dot\alpha\dot\beta}(\vec\sigma), \bar z_{\dot\gamma\dot\delta}(\vec\sigma')\}_{P.B.} = \tfrac{1}{2}(\delta^{\dot\alpha}_{\dot\gamma} \delta^{\dot\beta}_{\dot\delta} + \delta^{\dot\alpha}_{\dot\delta} \delta^{\dot\beta}_{\dot\gamma})\delta^p(\vec\sigma - \vec\sigma');\\ \{\pi^\alpha(\vec\sigma), \theta_\beta(\vec\sigma')\}_{P.B.} = \delta^\alpha_\beta \delta^p(\vec\sigma - \vec\sigma'), \quad \{\bar\pi^{\dot\alpha}(\vec\sigma), \bar\theta_{\dot\beta}(\vec\sigma')\}_{P.B.} = \delta^{\dot\alpha}_{\dot\beta} \delta^p(\vec\sigma - \vec\sigma');\\ \{P^\alpha_u(\vec\sigma), u_\beta(\vec\sigma')\}_{P.B.} = \delta^\alpha_\beta \delta^p(\vec\sigma - \vec\sigma'), \quad \{\bar P^{\dot\alpha}_u(\vec\sigma), \bar u_{\dot\beta}(\vec\sigma')\}_{P.B.} = \delta^{\dot\alpha}_{\dot\beta} \delta^p(\vec\sigma - \vec\sigma');\\ \{P^{(\rho)}_\mu(\vec\sigma), \rho^\nu(\vec\sigma')\}_{P.B.} = \delta^\nu_\mu \delta^p(\vec\sigma - \vec\sigma').\end{gathered} \tag{13}$$

The primary constraints characterizing the action (9) include Grassmannian Ψ-constraints

$$\begin{gathered}\Psi^\alpha = \pi^\alpha - 2i\bar\theta_{\dot\alpha} P^{\dot\alpha\alpha} - 4i\pi^{\alpha\beta}\theta_\beta \approx 0,\\ \bar\Psi^{\dot\alpha} = -(\Psi^\alpha)^* = \bar\pi^{\dot\alpha} - 2iP^{\dot\alpha\alpha}\theta_\alpha - 4i\bar\pi^{\dot\alpha\dot\beta}\bar\theta_{\dot\beta} \approx 0,\end{gathered} \tag{14}$$

together with bosonic Φ-constraints

$$\begin{aligned}\Phi^{\dot\alpha\alpha} &= P^{\dot\alpha\alpha} - \rho^\tau u^\alpha \bar u^{\dot\alpha} \approx 0,\\ \Phi^{\alpha\beta} &= \pi^{\alpha\beta} + \tfrac{1}{2}\rho^\tau u^\alpha u^\beta \approx 0,\\ \bar\Phi^{\dot\alpha\dot\beta} &= \bar\pi^{\dot\alpha\dot\beta} + \tfrac{1}{2}\rho^\tau \bar u^{\dot\alpha} \bar u^{\dot\beta} \approx 0\end{aligned} \tag{15}$$

added by the constraints coming from the sector of auxiliary world-volume fields $u_\alpha(\tau, \vec\sigma)$ and $\rho^\mu(\tau, \vec\sigma)$

$$P^\alpha_u \approx 0, \quad \bar P^{\dot\alpha}_u \approx 0, \tag{16}$$

$$P^{(\rho)}_\tau \approx 0, \quad P^{(\rho)}_M \approx 0, \tag{17}$$

where the world-volume index $\mu = (\tau, M)$ was split into the time-like and space-like $M = (1, 2, ..., p)$ subsets.

Using the definition of P.B. (13) it was found in [39] that the Ψ-constraints (14) have zero Poisson brackets with the primary bosonic constraints, but the P.B. of Ψ-constraints among themselves are non-zero

$$\begin{gathered}\{\Psi^\alpha(\vec\sigma), \Psi^\beta(\vec\sigma')\}_{P.B.} = -8i\pi^{\alpha\beta}\delta^p(\vec\sigma - \vec\sigma') = -8i\left(\Phi^{\alpha\beta} - \frac{1}{2}\rho^\tau u^\alpha u^\beta\right)\delta^p(\vec\sigma - \vec\sigma'),\\ \{\Psi^\alpha(\vec\sigma), \bar\Psi^{\dot\beta}(\vec\sigma')\}_{P.B.} = -4iP^{\dot\beta\alpha}\delta^p(\vec\sigma - \vec\sigma') = -4i\left(\Phi^{\dot\beta\alpha} + \rho^\tau u^\alpha \bar u^{\dot\beta}\right)\delta^p(\vec\sigma - \vec\sigma'),\\ \{\bar\Psi^{\dot\alpha}(\vec\sigma), \bar\Psi^{\dot\beta}(\vec\sigma')\}_{P.B.} = -8i\bar\pi^{\dot\alpha\dot\beta}\delta^p(\vec\sigma - \vec\sigma') = -8i\left(\bar\Phi^{\dot\alpha\dot\beta} - \frac{1}{2}\rho^\tau \bar u^{\dot\alpha} \bar u^{\dot\beta}\right)\delta^p(\vec\sigma - \vec\sigma').\end{gathered} \tag{18}$$

On the contrary the Φ-constraints (15) have zero Poisson brackets among themselves and with the Ψ−constraints (14)

$$\{\Phi(\vec\sigma), \Phi(\vec\sigma')\}_{P.B.} = 0, \quad \{\Phi(\vec\sigma), \Psi(\vec\sigma')\}_{P.B.} = 0, \tag{19}$$

but do not commute with P^{α}_u, $\bar{P}^{\dot\alpha}_u$

$$\begin{array}{rl}
\{P^{\alpha}_u(\vec\sigma), \Phi_{\beta\dot\gamma}(\vec\sigma')\}_{P.B.} = -\rho^{\tau}\delta^{\alpha}_{\beta}\bar{u}_{\dot\gamma}\delta^{p}(\vec\sigma-\vec\sigma'), \\
\{\bar{P}^{\dot\alpha}_u(\vec\sigma), \Phi_{\beta\dot\gamma}(\vec\sigma')\}_{P.B.} = -\rho^{\tau}\delta^{\dot\alpha}_{\dot\gamma}u_{\beta}\delta^{p}(\vec\sigma-\vec\sigma'), \\
\{P^{\alpha}_u(\vec\sigma), \Phi_{\beta\gamma}(\vec\sigma')\}_{P.B.} = \frac{1}{2}\rho^{\tau}(\delta^{\alpha}_{\beta}u_{\gamma}+\delta^{\alpha}_{\gamma}u_{\beta})\delta^{p}(\vec\sigma-\vec\sigma'), \\
\{\bar{P}^{\dot\alpha}_u(\vec\sigma), \bar\Phi_{\dot\beta\dot\gamma}(\vec\sigma')\}_{P.B.} = \frac{1}{2}\rho^{\tau}(\delta^{\dot\alpha}_{\dot\beta}\bar{u}_{\dot\gamma}+\delta^{\dot\alpha}_{\dot\gamma}\bar{u}_{\dot\beta})\delta^{p}(\vec\sigma-\vec\sigma')
\end{array} \tag{20}$$

and $P^{(\rho)}_{\tau}$ (16)

$$\begin{array}{l}
\{P^{(\rho)}_{\tau}(\vec\sigma), \Phi^{\beta\dot\gamma}(\vec\sigma')\}_{P.B.} = -u^{\beta}\bar{u}^{\dot\gamma}\delta^{p}(\vec\sigma-\vec\sigma'), \\
\{P^{(\rho)}_{\tau}(\vec\sigma), \Phi^{\alpha\beta}(\vec\sigma')\}_{P.B.} = \frac{1}{2}u^{\alpha}u^{\beta}\delta^{p}(\vec\sigma-\vec\sigma'), \\
\{P^{(\rho)}_{\tau}(\vec\sigma), \bar\Phi^{\dot\alpha\dot\beta}(\vec\sigma')\}_{P.B.} = \frac{1}{2}\bar{u}^{\dot\alpha}\bar{u}^{\dot\beta}\delta^{p}(\vec\sigma-\vec\sigma').
\end{array} \tag{21}$$

It was shown in [39] that there are 3 first-class constraints among Ψ-constraints and 6 first-class constraints among Φ-constraints. The momenta $P^{(\rho)}_{M} \approx 0$ also belong to the first class as it follows from the observation that the conjugate coordinates ρ^M do not enter the expressions for the primary constraints. Besides that there appear $(p+1)$ extra first-class constraints Δ_W and L_M corresponding to the world-volume Weyl and $\vec\sigma$-reparametrization gauge symmetries. The Weyl symmetry affects only auxiliary fields u_α and ρ^μ

$$\begin{array}{c}
\rho'^{\mu} = e^{-2\Lambda}\rho^{\mu}, \ u'_{\alpha} = e^{\Lambda}u_{\alpha}, \\
x'_{\alpha\dot\alpha} = x_{\alpha\dot\alpha}, \ z'_{\alpha\beta} = z_{\alpha\beta}, \ \theta'_{\alpha} = \theta_{\alpha}.
\end{array} \tag{22}$$

It is generated by the first-class constraint Δ_W

$$\Delta_W \equiv (P^{\alpha}_u u_{\alpha} + \bar{P}^{\dot\alpha}_u\bar{u}_{\dot\alpha}) - 2\rho^{\mu}P^{(\rho)}_{\mu} \approx 0. \tag{23}$$

The world-volume $\vec\sigma$-reparametrization constraints L_M are given by

$$\begin{array}{rl}
L_M = & P^{\dot\alpha\alpha}\partial_M x_{\alpha\dot\alpha} + \pi^{\alpha\beta}\partial_M z_{\alpha\beta} + \bar\pi^{\dot\alpha\dot\beta}\partial_M\bar{z}_{\dot\alpha\dot\beta} + \partial_M\theta_{\alpha}\pi^{\alpha} + \partial_M\bar\theta_{\dot\alpha}\bar\pi^{\dot\alpha} \\
+ & P^{\alpha}_u\partial_M u_{\alpha} + \bar{P}^{\dot\alpha}_u\partial_M\bar{u}_{\dot\alpha} - \partial_M P^{(\rho)}_{\tau}\rho^{\tau} - \partial_M P^{(\rho)}_{N}\rho^{N} \approx 0.
\end{array} \tag{24}$$

They are secondary constraints resulting from the Dirac selfconsistency procedure.

The Hamiltonian structure formed by the above defined constraints after their covariant division into the first and second classes, as well as the corresponding Dirac brackets (D.B.) were studied in [39]. The procedure of the covariant constraint division used in [39] involved additional auxiliary spinor field $v_\alpha(\tau,\vec\sigma)$ forming together with $u_\alpha(\tau,\vec\sigma)$ local spinorial basis attached to string/brane world-volume. As a result, the additional first-class constraints $P^{(u)}_v$ and $\bar{P}^{(u)}_v$

$$P^{(u)}_v \equiv P^{\alpha}_v u_{\alpha} \approx 0, \quad \bar{P}^{(u)}_v \equiv \bar{P}^{\dot\alpha}_v\bar{u}_{\dot\alpha} \approx 0 \tag{25}$$

arose corresponding to the gauge shifts of v_α along u_α

$$\delta_\epsilon v_\alpha = \epsilon u_\alpha, \quad \delta_\epsilon \bar{v}_{\dot\alpha} = \bar\epsilon\bar{u}_{\dot\alpha}. \tag{26}$$

These shifts preserve the linear independence condition $u^\alpha v_\alpha = 1$ of the basis spinors u^α, v^α and their D.B. turns out to be quadratic in the first class constraints $\tilde{T}^{(\pm)}$, $\Psi^{(u)}$ and its c.c.

$$\begin{array}{rl}
\{P^{(u)}_v(\vec\sigma), \bar{P}^{(u)}_v(\vec\sigma')\}_{D.B.} = & \frac{1}{4\rho^{\tau}}[\tilde{T}^{(+)}(\bar{P}^{(u)}_v - P^{(u)}_v) + \tilde{T}^{(-)}(\bar{P}^{(u)}_v + P^{(u)}_v) \\
+ & \frac{1}{4}\Psi^{(u)}\bar\Psi^{(u)}]\delta^{p}(\vec\sigma-\vec\sigma').
\end{array} \tag{27}$$

This shows that the D.B. algebra of the first-class constraints has the rank equal two and it gives rise to the higher powers of the ghosts in the BRST generator similarly to the (super) p-brane theory without TCC coordinates [43]. By the analogy the quadratic terms in the first-class constraints appear in the r.h.s. of D.B's of the $OSp(1|8)$ superalgebra.

Such a non-linear character of the both algebra of constraints and global $OSp(1|8)$ supersymmetry algebra complicates transition to the quantum theory. It gives a reason to apply the conversion method [40]-[42] transforming all the primary and secondary constraints in the first class. This conversion method might turn out to be more efficient, similarly to the case of (super)particles [44]-[47], and we will utilize it here.

In the next Section we consider conversion of the primary and secondary constraints (14)-(17), (23), (24) to the first class and examine quantum realization of the symmetries of the model under question.

3 Conversion of the primary and secondary constraints to the first class

To convert the constraints (16) and (17) to the first class we introduce new variables $\tilde{u}^\alpha = u^\alpha - q^\alpha$ and $\tilde{\rho}^\tau = \rho^\tau - \varphi^\tau$, where q^α and φ^τ are auxiliary conversion variables canonically conjugate to momenta P_q^α and $P_\tau^{(\varphi)}$

$$\{P_q^\alpha(\vec{\sigma}), q_\beta(\vec{\sigma}')\}_{P.B.} = \delta^\alpha_\beta \delta^p(\vec{\sigma} - \vec{\sigma}'), \quad \{\bar{P}_q^{\dot{\alpha}}(\vec{\sigma}), \bar{q}_{\dot{\beta}}(\vec{\sigma}')\}_{P.B.} = \delta^{\dot{\alpha}}_{\dot{\beta}} \delta^p(\vec{\sigma} - \vec{\sigma}'),$$
$$\{P_\tau^{(\varphi)}(\vec{\sigma}), \varphi^\tau(\vec{\sigma}')\}_{P.B.} = \delta^p(\vec{\sigma} - \vec{\sigma}'). \tag{28}$$

In terms of new momenta the converted constraints (16) and (17) are presented as

$$\widetilde{P}_u^\alpha = P_u^\alpha + P_q^\alpha \approx 0, \quad \bar{\widetilde{P}}_u^{\dot{\alpha}} = \bar{P}_u^{\dot{\alpha}} + \bar{P}_q^{\dot{\alpha}} \approx 0, \tag{29}$$

$$\widetilde{P}_\tau^{(\rho)} = P_\tau^{(\rho)} + P_\tau^{(\varphi)} \approx 0,$$
$$P_M^{(\rho)} \approx 0 \tag{30}$$

after the correspondent modifications of the $\Phi-$ and Ψ- sector constraints. The converted bosonic constraints $\widetilde{\Phi} \equiv (\widetilde{\Phi}^{\dot{\alpha}\alpha}, \widetilde{\Phi}^{\alpha\beta}, \bar{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}})$ originating from the Φ-constraints (15) acquire the form

$$\widetilde{\Phi}^{\dot{\alpha}\alpha} = P^{\dot{\alpha}\alpha} - \tilde{\rho}^\tau \tilde{u}^\alpha \bar{\tilde{u}}^{\dot{\alpha}} \approx 0,$$
$$\widetilde{\Phi}^{\alpha\beta} = \pi^{\alpha\beta} + \tfrac{1}{2} \tilde{\rho}^\tau \tilde{u}^\alpha \tilde{u}^\beta \approx 0, \tag{31}$$
$$\bar{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}} = \bar{\pi}^{\dot{\alpha}\dot{\beta}} + \tfrac{1}{2} \tilde{\rho}^\tau \bar{\tilde{u}}^{\dot{\alpha}} \bar{\tilde{u}}^{\dot{\beta}} \approx 0,$$

and have zero P.B. with the converted constraints (29), (30) and among themselves. Then the converted fermionic constraints $\widetilde{\Psi} = (\widetilde{\Psi}^\alpha, \bar{\widetilde{\Psi}}^{\dot{\alpha}})$ originating from the Ψ-constraints (14) take the form

$$\widetilde{\Psi}^\alpha = \pi^\alpha - 2i\bar{\theta}_{\dot{\alpha}} P^{\dot{\alpha}\alpha} - 4i\pi^{\alpha\beta}\theta_\beta + 2(\tilde{\rho}^\tau)^{1/2} \tilde{u}^\alpha f \approx 0,$$
$$\bar{\widetilde{\Psi}}^{\dot{\alpha}} = -(\Psi^\alpha)^* = \bar{\pi}^{\dot{\alpha}} - 2iP^{\dot{\alpha}\alpha}\theta_\alpha - 4i\bar{\pi}^{\dot{\alpha}\dot{\beta}}\bar{\theta}_{\dot{\beta}} - 2(\tilde{\rho}^\tau)^{1/2} \bar{\tilde{u}}^{\dot{\alpha}} f \approx 0, \tag{32}$$

where $f^* = f$ is an auxiliary Grassmannian variable characterized by the P.B.

$$\{f(\vec{\sigma}), f(\vec{\sigma}')\}_{P.B.} = -i\delta^p(\vec{\sigma} - \vec{\sigma}'). \tag{33}$$

The addition of f restores the forth κ-symmetry and transforms the $\widetilde{\Psi}$-constraints to the first class leading to non-zero P.B. only among themselves

$$\{\widetilde{\Psi}^{\alpha}(\vec{\sigma}), \widetilde{\Psi}^{\beta}(\vec{\sigma}')\}_{P.B.} = -8i\widetilde{\Phi}^{\alpha\beta}\delta^p(\vec{\sigma}-\vec{\sigma}'), \tag{34}$$

$$\{\widetilde{\Psi}^{\alpha}(\vec{\sigma}), \bar{\widetilde{\Psi}}^{\dot{\beta}}(\vec{\sigma}')\}_{P.B.} = -4i\widetilde{\Phi}^{\dot{\beta}\alpha}\delta^p(\vec{\sigma}-\vec{\sigma}'). \tag{35}$$

The Weyl symmetry (23) and world-volume $\vec{\sigma}-$reparametrization constraints in the extended phase space are given by

$$\widetilde{\Delta}_W \equiv (\tilde{\tilde{P}}{}^{\alpha}_u\tilde{u}_{\alpha} + \bar{\tilde{\tilde{P}}}{}^{\dot{\alpha}}_u\bar{\tilde{u}}_{\dot{\alpha}}) - 2\tilde{\rho}^{\tau}\tilde{\tilde{P}}{}^{(\rho)}_{\tau} - 2\rho^M P^{(\rho)}_M \approx 0, \tag{36}$$

where

$$\tilde{\tilde{P}}{}^{\alpha}_u = \frac{1}{2}(P^{\alpha}_u - P^{\alpha}_q), \qquad \tilde{\tilde{P}}{}^{(\rho)}_{\tau} = \frac{1}{2}(P^{(\rho)}_{\tau} - P^{(\varphi)}_{\tau}), \tag{37}$$

and respectively

$$\begin{aligned} \widetilde{L}_M = \;& P^{\dot{\alpha}\alpha}\partial_M x_{\alpha\dot{\alpha}} + \pi^{\alpha\beta}\partial_M z_{\alpha\beta} + \bar{\pi}^{\dot{\alpha}\dot{\beta}}\partial_M \bar{z}_{\dot{\alpha}\dot{\beta}} + \partial_M\theta_{\alpha}\pi^{\alpha} + \partial_M\bar{\theta}_{\dot{\alpha}}\bar{\pi}^{\dot{\alpha}} \\ +\;& (\tilde{\tilde{P}}{}^{\alpha}_u\partial_M\tilde{u}_{\alpha} + \bar{\tilde{\tilde{P}}}{}^{\dot{\alpha}}_u\partial_M\bar{\tilde{u}}_{\dot{\alpha}}) \\ -\;& \partial_M\tilde{\tilde{P}}{}^{(\rho)}_{\tau}\tilde{\rho}^{\tau} - \partial_M P^{(\rho)}_N\rho^N - \tfrac{i}{2}f\partial_M f \approx 0. \end{aligned} \tag{38}$$

These converted constraints satisfy the following non zero P.B. relations

$$\{\widetilde{\Delta}_W(\vec{\sigma}), P^{(\rho)}_M(\vec{\sigma}')\}_{P.B.} = 2P^{(\rho)}_M\delta^p(\vec{\sigma}-\vec{\sigma}'), \tag{39}$$

$$\{\widetilde{L}_M(\vec{\sigma}), \widetilde{P}^{\alpha}_u(\vec{\sigma}')\}_{P.B.} = 0, \tag{40}$$

$$\{\widetilde{L}_M(\vec{\sigma}), \widetilde{P}^{(\rho)}_{\tau}(\vec{\sigma}')\}_{P.B.} = 0, \tag{41}$$

$$\{\widetilde{L}_M(\vec{\sigma}), P^{(\rho)}_N(\vec{\sigma}')\}_{P.B.} = \partial_M\tilde{P}^{(\rho)}_N\delta^p(\vec{\sigma}-\vec{\sigma}'), \tag{42}$$

$$\{\widetilde{L}_M(\vec{\sigma}), \widetilde{L}_N(\vec{\sigma}')\}_{P.B.} = (\widetilde{L}_M(\vec{\sigma}')\partial_{N'} - \widetilde{L}_N(\vec{\sigma})\partial_M)\delta^p(\vec{\sigma}-\vec{\sigma}'), \tag{43}$$

$$\{\widetilde{L}_M(\vec{\sigma}), [\text{other constraints}](\vec{\sigma}')\}_{P.B.} = -[\text{other constraints}](\vec{\sigma})\partial_M\delta^p(\vec{\sigma}-\vec{\sigma}') \tag{44}$$

and the complex conjugate ones. Other P.B's. are equal to zero in the strong sense.

One can easily verify that there was added such the number of conversion variables so that the number of physical degrees of freedom in the enlarged phase space is equal to

$$\begin{aligned} n^{\Phi}_{\text{phys}} =\;& 2(15_b + 4_f)_{\text{original}} + 2(5_b + \tfrac{1}{2}1_f)_{\text{conversion}} - 2((16+p)_b + 4_f)_{1^{\underline{st}}\text{ cl. constr.}} \\ =\;& 2(4-p)_b + 1_f, \end{aligned} \tag{45}$$

and it coincides with the number of physical degrees of freedom in the original model

$$\begin{aligned} n_{\text{phys}} =\;& 2(15_b + 4_f)_{\text{original}} - 2((7+p)_b + 3_f)_{1^{\underline{st}}\text{ cl. constr.}} - (8_b + 1_f)_{2^{\underline{nd}}\text{ cl. constr.}} \\ =\;& 2(4-p)_b + 1_f. \end{aligned} \tag{46}$$

Note that we have not taken into account the variables ρ_M which are involved in the original and extended phase spaces, because of the first-class constraints $P^{(\rho)}_M$ (30) presence in both of these phase spaces. Thus, we have solved the conversion problem and found the first-class constraints in the extended phase space. The next step is to find a realization of the $OSp(1|8)$ global supersymmetry generators in the extended phase space.

4 Generators of the $OSp(1|8)$ global supersymmetry in the extended phase space

Here we shall construct the $OSp(1|8)$ generators in the extended phase space based on their representation built in [39]. The key principle to find these extended generators is the requirement for them to form a closed P.B. algebra with the above considered converted first-class constraints. This requirement together with the requirement to generate $OSp(1|8)$ superalgebra in the extended phase space will uniquely restore the form of these generators.

We find that the generalized translation generators together the generator densities Q^{α} and $Q^{\dot\alpha}$ of the $N=1$ global supersymmetry remain unchanged upon transition to extended phase space

$$\begin{aligned} Q^{\alpha}(\tau,\vec\sigma) &= \pi^{\alpha} + 2i\bar\theta_{\dot\alpha}P^{\dot\alpha\alpha} + 4i\pi^{\alpha\beta}\theta_{\beta}, \\ \bar Q^{\dot\alpha}(\tau,\vec\sigma) &= \bar\pi^{\dot\alpha} + 2iP^{\dot\alpha\alpha}\theta_{\alpha} + 4i\bar\pi^{\dot\alpha\dot\beta}\bar\theta_{\dot\beta} \end{aligned} \tag{47}$$

and their P.B's. have the standard form

$$\begin{aligned} \{Q^{\alpha}(\vec\sigma),\bar Q^{\dot\alpha}(\vec\sigma')\}_{P.B.} &= 4iP^{\dot\alpha\alpha}\delta^{p}(\vec\sigma-\vec\sigma'), \\ \{Q^{\alpha}(\vec\sigma),Q^{\beta}(\vec\sigma')\}_{P.B.} &= 8i\pi^{\alpha\beta}\delta^{p}(\vec\sigma-\vec\sigma'). \end{aligned} \tag{48}$$

To build the extended "square roots" $\widetilde S_{\gamma}$ and $\bar{\widetilde S}_{\dot\gamma}$ of the generalized conformal boost densities we consider the P.B. relations $\{\widetilde\Psi^{\alpha}(\vec\sigma),S_{\gamma}(\vec\sigma')\}_{P.B.}$ and find that the additional term $\frac{2}{(\tilde\rho^{\tau})^{1/2}}\tilde{\tilde P}_{u\gamma}f$ has to be added to S_{γ} to close the P.B.

$$\{\widetilde\Psi^{\alpha}(\vec\sigma),\widetilde S_{\gamma}(\sigma')\}_{P.B.} = 4i\delta^{\alpha}_{\gamma}[\widetilde\Psi^{\beta}\theta_{\beta} + \bar{\widetilde\Psi}^{\dot\beta}\bar\theta_{\dot\beta}]\delta^{p}(\vec\sigma-\vec\sigma'), \tag{49}$$

where $\widetilde S_{\gamma}$ and its complex conjugate are given by

$$\begin{aligned} \widetilde S_{\gamma}(\tau,\vec\sigma) &= z_{\gamma\delta}Q^{\delta} + x_{\gamma\dot\delta}\bar Q^{\dot\delta} - 2i\theta_{\gamma}(\theta_{\delta}\pi^{\delta} + \bar\theta_{\dot\delta}\bar\pi^{\dot\delta}) + 4i(\tilde u^{\delta}\theta_{\delta} - \bar{\tilde u}^{\dot\delta}\bar\theta_{\dot\delta})\tilde{\tilde P}_{u\gamma} + \tfrac{2}{(\tilde\rho^{\tau})^{1/2}}\tilde{\tilde P}_{u\gamma}f, \\ \bar{\widetilde S}_{\dot\gamma}(\tau,\vec\sigma) &= \bar z_{\dot\gamma\dot\delta}\bar Q^{\dot\delta} + x_{\delta\dot\gamma}Q^{\delta} - 2i\bar\theta_{\dot\gamma}(\theta_{\delta}\pi^{\delta} + \bar\theta_{\dot\delta}\bar\pi^{\dot\delta}) - 4i(\tilde u^{\delta}\theta_{\delta} - \bar{\tilde u}^{\dot\delta}\bar\theta_{\dot\delta})\bar{\tilde{\tilde P}}_{u\dot\gamma} - \tfrac{2}{(\tilde\rho^{\tau})^{1/2}}\bar{\tilde{\tilde P}}_{u\dot\gamma}f. \end{aligned} \tag{50}$$

The representations (50) can be used to find the generalized conformal boost densities $\widetilde K_{\gamma\lambda}$ and $\widetilde K_{\gamma\dot\gamma}$ from the known P.B. relations of $OSp(1|8)$ superalgebra

$$\{\widetilde S_{\gamma}(\vec\sigma),\widetilde S_{\lambda}(\vec\sigma')\}_{P.B.} = 4i\widetilde K_{\gamma\lambda}\delta^{p}(\vec\sigma-\vec\sigma'), \quad \{\widetilde S_{\gamma}(\vec\sigma),\bar{\widetilde S}_{\dot\gamma}(\vec\sigma')\}_{P.B.} = 4i\widetilde K_{\gamma\dot\gamma}\delta^{p}(\vec\sigma-\vec\sigma'), \tag{51}$$

resulting in the explicit form for the $K-$generators

$$\begin{aligned} \widetilde K_{\gamma\lambda}(\tau,\vec\sigma) = {} & 2z_{\gamma\beta}z_{\lambda\delta}\pi^{\beta\delta} + 2x_{\gamma\dot\beta}x_{\lambda\dot\delta}\bar\pi^{\dot\beta\dot\delta} + z_{\gamma\beta}x_{\lambda\dot\delta}P^{\dot\delta\beta} + x_{\gamma\dot\beta}z_{\lambda\delta}P^{\dot\beta\delta} \\ + {} & \theta_{\lambda}(z_{\gamma\delta}\pi^{\delta} + x_{\gamma\dot\delta}\bar\pi^{\dot\delta}) + \theta_{\gamma}(z_{\lambda\delta}\pi^{\delta} + x_{\lambda\dot\beta}\bar\pi^{\dot\delta}) \\ + {} & (\tilde u^{\delta}z_{\delta\lambda} - \bar{\tilde u}^{\dot\delta}x_{\lambda\dot\delta})\tilde{\tilde P}_{u\gamma} + (\tilde u^{\delta}z_{\delta\gamma} - \bar{\tilde u}^{\dot\delta}x_{\gamma\dot\delta})\tilde{\tilde P}_{u\lambda} \\ - {} & 2i(\tilde u^{\delta}\theta_{\delta} - \bar{\tilde u}^{\dot\delta}\bar\theta_{\dot\delta})(\theta_{\lambda}\tilde{\tilde P}_{u\gamma} + \theta_{\gamma}\tilde{\tilde P}_{u\lambda}) \\ + {} & \tfrac{2}{(\tilde\rho^{\tau})^{1/2}}(\theta_{\lambda}\tilde{\tilde P}_{u\gamma} + \theta_{\gamma}\tilde{\tilde P}_{u\lambda})f - \tfrac{1}{\tilde\rho^{\tau}}\tilde{\tilde P}_{u\gamma}\tilde{\tilde P}_{u\lambda}, \end{aligned} \tag{52}$$

$$
\begin{aligned}
\widetilde{K}_{\gamma\dot\gamma}(\tau,\vec\sigma) = \ & z_{\gamma\delta}\bar z_{\dot\gamma\dot\delta}P^{\dot\delta\delta} + x_{\gamma\dot\delta}x_{\delta\dot\gamma}P^{\dot\delta\delta} + 2(z_{\gamma\delta}x_{\lambda\dot\gamma}\pi^{\delta\lambda} + x_{\gamma\dot\delta}\bar z_{\dot\gamma\dot\lambda}\bar\pi^{\dot\delta\dot\lambda}) \\
+ \ & \theta_\gamma(\bar z_{\dot\gamma\dot\delta}\bar\pi^{\dot\delta} + x_{\delta\dot\gamma}\pi^\delta) + \bar\theta_{\dot\gamma}(z_{\gamma\delta}\pi^\delta + x_{\gamma\dot\delta}\bar\pi^{\dot\delta}) \\
+ \ & (\tilde u^\delta x_{\delta\dot\gamma} - \bar{\tilde u}^{\dot\delta}\bar z_{\dot\delta\dot\gamma})\tilde{\tilde P}_{u\gamma} + (x_{\gamma\dot\delta}\bar{\tilde u}^{\dot\delta} - z_{\gamma\delta}\tilde u^\delta)\bar{\tilde{\tilde P}}_{u\dot\gamma} \\
- \ & 2i(\tilde u^\delta\theta_\delta - \bar{\tilde u}^{\dot\delta}\bar\theta_{\dot\delta})(\bar\theta_{\dot\gamma}\tilde{\tilde P}_{u\gamma} - \theta_\gamma\bar{\tilde{\tilde P}}_{u\dot\gamma}) \\
+ \ & \tfrac{2}{(\tilde\rho^\tau)^{1/2}}(\bar\theta_{\dot\gamma}\tilde{\tilde P}_{u\gamma} - \theta_\gamma\bar{\tilde{\tilde P}}_{u\dot\gamma})f + \tfrac{1}{\tilde\rho^\tau}\tilde{\tilde P}_{u\gamma}\bar{\tilde{\tilde P}}_{u\dot\gamma}.
\end{aligned}
\tag{53}
$$

The correctness of the representations (50) is verified by the reproducibility of the known P.B. relations

$$
\{Q_\alpha(\vec\sigma),\widetilde{K}_{\beta\gamma}(\vec\sigma')\}_{P.B.} = (\varepsilon_{\alpha\beta}\widetilde{S}_\gamma + \varepsilon_{\alpha\gamma}\widetilde{S}_\beta)\delta^p(\vec\sigma-\vec\sigma'),\ \{Q_\alpha(\vec\sigma),\widetilde{K}_{\beta\dot\gamma}(\vec\sigma')\}_{P.B.} = \varepsilon_{\alpha\beta}\bar{\widetilde{S}}_{\dot\gamma}\delta^p(\vec\sigma-\vec\sigma'). \tag{54}
$$

Remaining 16 generator densities of the $Sp(8)$ algebra $\widetilde{L}^\alpha{}_\beta$, $\widetilde{L}^\alpha{}_{\dot\beta}$ that include the Lorentz symmetry generator densities are given by the expressions

$$
\begin{aligned}
\widetilde{L}^\alpha{}_\beta(\tau,\vec\sigma) &= P^{\dot\beta\alpha}x_{\beta\dot\beta} + 2\pi^{\alpha\gamma}z_{\gamma\beta} - \pi^\alpha\theta_\beta + \tilde u^\alpha\tilde{\tilde P}_{u\beta}, \\
\widetilde{L}^\alpha{}_{\dot\beta}(\tau,\vec\sigma) &= 2\pi^{\alpha\gamma}x_{\gamma\dot\beta} + P^{\dot\gamma\alpha}\bar z_{\dot\beta\dot\gamma} - \pi^\alpha\bar\theta_{\dot\beta} - \tilde u^\alpha\bar{\tilde{\tilde P}}_{u\dot\beta}
\end{aligned}
\tag{55}
$$

and their complex conjugate, which follow from the P.B. relations of the $OSp(1|8)$ superalgebra

$$
\{Q_\alpha(\vec\sigma),\widetilde{S}_\beta(\vec\sigma')\}_{P.B.} = 4i\widetilde{L}_{\alpha\beta}\delta^p(\vec\sigma-\vec\sigma'),\quad \{Q_\alpha(\vec\sigma),\bar{\widetilde{S}}_{\dot\beta}(\vec\sigma')\}_{P.B.} = 4i\widetilde{L}_{\alpha\dot\beta}\delta^p(\vec\sigma-\vec\sigma'), \tag{56}
$$

where the representations (47) and (50) have been used.

The correctness of the representations (55) is verified by the reproducibility of well known P.B's. for the L-densities

$$
\begin{gathered}
\{\widetilde{L}_{\alpha\beta}(\vec\sigma),\widetilde{L}_{\gamma\delta}(\vec\sigma')\}_{P.B.} = (\varepsilon_{\beta\gamma}\widetilde{L}_{\alpha\delta} + \varepsilon_{\alpha\delta}\widetilde{L}_{\gamma\beta})\delta^p(\vec\sigma-\vec\sigma'), \\
\{\widetilde{L}_{\alpha\beta}(\vec\sigma),\widetilde{L}_{\dot\gamma\delta}(\vec\sigma')\}_{P.B.} = \varepsilon_{\alpha\delta}\widetilde{L}_{\dot\gamma\beta}\delta^p(\vec\sigma-\vec\sigma'),\quad \{\widetilde{L}_{\alpha\beta}(\vec\sigma),\widetilde{L}_{\gamma\dot\delta}(\vec\sigma')\}_{P.B.} = \varepsilon_{\beta\gamma}\widetilde{L}_{\alpha\dot\delta}\delta^p(\vec\sigma-\vec\sigma'), \\
\{\widetilde{L}_{\alpha\dot\beta}(\vec\sigma),\widetilde{L}_{\dot\gamma\delta}(\vec\sigma')\}_{P.B.} = (\varepsilon_{\dot\beta\dot\gamma}\widetilde{L}_{\alpha\delta} + \varepsilon_{\alpha\delta}\bar{\widetilde{L}}_{\dot\gamma\dot\beta})\delta^p(\vec\sigma-\vec\sigma')
\end{gathered}
\tag{57}
$$

added by their P.B's. with the densities of supercharges Q and $\widetilde{S}$

$$
\begin{gathered}
\{Q_\alpha(\vec\sigma),\widetilde{L}_{\beta\gamma}(\vec\sigma')\}_{P.B.} = \varepsilon_{\alpha\gamma}Q_\beta\delta^p(\vec\sigma-\vec\sigma'),\quad \{Q_\alpha(\vec\sigma),\widetilde{L}^{\dot\beta}{}_\gamma(\vec\sigma')\}_{P.B.} = -\varepsilon_{\alpha\gamma}\bar Q^{\dot\beta}\delta^p(\vec\sigma-\vec\sigma'), \\
\{\widetilde{S}_\alpha(\vec\sigma),\widetilde{L}_{\beta\gamma}(\vec\sigma')\}_{P.B.} = \varepsilon_{\alpha\beta}\widetilde{S}_\gamma\delta^p(\vec\sigma-\vec\sigma'),\quad \{\widetilde{S}_\alpha(\vec\sigma),\widetilde{L}_\beta{}^{\dot\gamma}(\vec\sigma')\}_{P.B.} = \varepsilon_{\alpha\beta}\bar{\widetilde{S}}^{\dot\gamma}\delta^p(\vec\sigma-\vec\sigma')
\end{gathered}
\tag{58}
$$

and with the conformal boost densities $\widetilde{K}$

$$
\begin{gathered}
\{\widetilde{L}_{\alpha\beta}(\vec\sigma),\widetilde{K}_{\gamma\delta}(\vec\sigma')\}_{P.B.} = (\varepsilon_{\alpha\gamma}\widetilde{K}_{\beta\delta} + \varepsilon_{\alpha\delta}\widetilde{K}_{\beta\gamma})\delta^p(\vec\sigma-\vec\sigma'), \\
\{L_{\alpha\beta}(\vec\sigma),\widetilde{K}_{\gamma\dot\delta}(\vec\sigma')\}_{P.B.} = \varepsilon_{\alpha\gamma}\widetilde{K}_{\beta\dot\delta}\delta^p(\vec\sigma-\vec\sigma'), \\
\{\widetilde{L}_{\alpha\dot\beta}(\vec\sigma),\widetilde{K}_{\gamma\delta}(\vec\sigma')\}_{P.B.} = (\varepsilon_{\alpha\gamma}\widetilde{K}_{\delta\dot\beta} + \varepsilon_{\alpha\delta}\widetilde{K}_{\gamma\dot\beta})\delta^p(\vec\sigma-\vec\sigma'), \\
\{\widetilde{L}_{\alpha\dot\beta}(\vec\sigma),\widetilde{K}_{\gamma\dot\delta}(\vec\sigma')\}_{P.B.} = \varepsilon_{\alpha\gamma}\bar{\widetilde{K}}_{\dot\beta\dot\delta}\delta^p(\vec\sigma-\vec\sigma'),
\end{gathered}
\tag{59}
$$

where we adduced only nonzero Poisson brackets.

The remaining nonzero P.B's. of the considered orthosymplectic superalgebra include the generator densities $P^{\beta\dot{\gamma}}$, $\pi^{\beta\gamma}$ of the generalized translations

$$
\begin{gathered}
\{\widetilde{S}_\alpha(\vec{\sigma}), \pi_\beta{}^\gamma(\vec{\sigma}')\}_{P.B.} = -\tfrac{1}{2}(\delta^\gamma_\alpha Q_\beta - \varepsilon_{\alpha\beta}Q^\gamma)\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{\widetilde{S}_\alpha(\vec{\sigma}), P_{\beta\dot{\gamma}}(\vec{\sigma}')\}_{P.B.} = -\varepsilon_{\alpha\beta}\bar{Q}_{\dot{\gamma}}\delta^p(\vec{\sigma}-\vec{\sigma}');\\
\{\pi_\alpha{}^\beta(\vec{\sigma}), \widetilde{L}_{\gamma\delta}(\vec{\sigma}')\}_{P.B.} = (\delta^\beta_\delta \pi_{\alpha\gamma} + \varepsilon_{\alpha\delta}\pi_\gamma{}^\beta)\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{\pi_\alpha{}^\beta(\vec{\sigma}), \widetilde{L}_{\dot{\gamma}\delta}(\vec{\sigma}')\}_{P.B.} = \tfrac{1}{2}(\delta^\beta_\delta P_{\alpha\dot{\gamma}} - \varepsilon_{\alpha\delta}P^\beta{}_{\dot{\gamma}})\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{P_{\alpha\dot{\beta}}(\vec{\sigma}), \widetilde{L}_{\gamma\delta}(\vec{\sigma}')\}_{P.B.} = \varepsilon_{\alpha\delta}P_{\gamma\dot{\beta}}\delta^p(\vec{\sigma}-\vec{\sigma}'),\ \{P_{\alpha\dot{\beta}}(\vec{\sigma}), \widetilde{L}^{\dot{\gamma}}{}_\delta(\vec{\sigma}')\}_{P.B.} = 2\varepsilon_{\alpha\delta}\bar{\pi}^{\dot{\gamma}}{}_{\dot{\beta}}\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{\pi_\alpha{}^\beta(\vec{\sigma}), \widetilde{K}_{\gamma\delta}(\vec{\sigma}')\}_{P.B.} = \tfrac{1}{2}(\delta^\beta_\gamma \widetilde{L}_{\alpha\delta} + \varepsilon_{\alpha\gamma}\widetilde{L}^\beta{}_\delta + \delta^\delta_\beta \widetilde{L}_{\alpha\gamma} + \varepsilon_{\alpha\delta}\widetilde{L}^\beta{}_\gamma)\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{\pi_\alpha{}^\beta(\vec{\sigma}), \widetilde{K}_{\gamma\dot{\delta}}(\vec{\sigma}')\}_{P.B.} = -\tfrac{1}{2}(\delta^\beta_\gamma \widetilde{L}_{\alpha\dot{\delta}} - \varepsilon_{\alpha\gamma}\widetilde{L}^\beta{}_{\dot{\delta}})\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{P_{\alpha\dot{\beta}}(\vec{\sigma}), \widetilde{K}_{\gamma\delta}(\vec{\sigma}')\}_{P.B.} = (\varepsilon_{\alpha\delta}\widetilde{L}_{\dot{\beta}\gamma} + \varepsilon_{\alpha\gamma}\widetilde{L}_{\dot{\beta}\delta})\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{P_{\alpha\dot{\beta}}(\vec{\sigma}), \widetilde{K}_{\gamma\dot{\delta}}(\vec{\sigma}')\}_{P.B.} = (\varepsilon_{\alpha\gamma}\bar{\widetilde{L}}_{\dot{\beta}\dot{\delta}} + \varepsilon_{\dot{\beta}\dot{\delta}}\widetilde{L}_{\alpha\gamma})\delta^p(\vec{\sigma}-\vec{\sigma}').
\end{gathered}
\tag{60}
$$

These expressions should be complemented by their complex conjugate.

So, one can conclude that the above considered P.B's. correctly reproduce the well known P.B. commutation relations of the $OSp(1|8)$ superalgebra and, therefore, the problem of realization of this superalgebra in the extended phase space of the model is solved.

The next step is to find the P.B. commutation relations between the generators of the $OSp(1|8)$ superalgebra and the generators of the gauge symmetry of the model presented by the first-class constraints constructed in the Section 3.

5 The P.B. superalgebra of global and gauge symmetries in the extended phase space

Taking into account that the $OSp(1|8)$ superalgebra generators are identified with the physical observables their P.B. relations with the converted first-class constraints have to be zero in the weak sense. Fulfilment of this condition would prove the gauge invariant character of the considered super p-brane model. In this section we show that this condition is actually satisfied on the classical level of the P.B. superalgebra.

At first, we find that the $\widetilde{\Phi}$-constraints have the following nonzero Poisson brackets with the $\widetilde{L}$ generator densities of $OSp(1|8)$

$$
\begin{gathered}
\{\widetilde{\Phi}^{\alpha\beta}(\vec{\sigma}), \widetilde{L}_{\gamma\delta}(\vec{\sigma}')\}_{P.B.} = (\delta^\alpha_\delta \widetilde{\Phi}_\gamma{}^\beta + \delta^\beta_\delta \widetilde{\Phi}_\gamma{}^\alpha)\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{\widetilde{\Phi}^{\alpha\beta}(\vec{\sigma}), L_{\dot{\gamma}\delta}(\vec{\sigma}')\}_{P.B.} = \tfrac{1}{2}(\delta^\alpha_\delta \widetilde{\Phi}^\beta{}_{\dot{\gamma}} + \delta^\beta_\delta \widetilde{\Phi}^\alpha{}_{\dot{\gamma}})\delta^p(\vec{\sigma}-\vec{\sigma}');\\
\{\widetilde{\Phi}^{\dot{\alpha}\alpha}(\vec{\sigma}), L_{\gamma\delta}(\vec{\sigma}')\}_{P.B.} = \delta^\alpha_\delta \widetilde{\Phi}_\gamma{}^{\dot{\alpha}}\delta^p(\vec{\sigma}-\vec{\sigma}'),\\
\{\widetilde{\Phi}^{\dot{\alpha}\alpha}(\vec{\sigma}), L_{\gamma\dot{\delta}}(\vec{\sigma}')\}_{P.B.} = 2\delta^{\dot{\alpha}}_{\dot{\delta}} \widetilde{\Phi}_\gamma{}^\alpha\delta^p(\vec{\sigma}-\vec{\sigma}')
\end{gathered}
\tag{61}
$$

and conclude that the r.h.s. of (61) are zero in the weak sense.

The same conclusion is satisfied for both the nonzero P.B. with $\widetilde{S}$ generator densities

$$\begin{aligned}\{\widetilde{\Phi}^{\alpha\beta}(\vec{\sigma}),\widetilde{S}_{\gamma}(\vec{\sigma}')\}_{P.B.} = &\ \tfrac{1}{2}[\delta^{\alpha}_{\gamma}(\widetilde{\Psi}^{\beta}+8i\widetilde{\Phi}^{\beta\delta}\theta_{\delta}+4i\widetilde{\Phi}^{\dot{\delta}\beta}\bar{\theta}_{\dot{\delta}}) \\ + &\ \delta^{\beta}_{\gamma}(\widetilde{\Psi}^{\alpha}+8i\widetilde{\Phi}^{\alpha\delta}\theta_{\delta}+4i\widetilde{\Phi}^{\dot{\delta}\alpha}\bar{\theta}_{\dot{\delta}})]\delta^{p}(\vec{\sigma}-\vec{\sigma}'); \\ \{\widetilde{\Phi}^{\dot{\alpha}\alpha}(\vec{\sigma}),S_{\gamma}(\vec{\sigma}')\}_{P.B.} = &\ \delta^{\alpha}_{\gamma}(\bar{\widetilde{\Psi}}^{\dot{\alpha}}+8i\bar{\widetilde{\Phi}}^{\dot{\alpha}\dot{\delta}}\bar{\theta}_{\dot{\delta}}+4i\widetilde{\Phi}^{\dot{\alpha}\delta}\theta_{\delta})\delta^{p}(\vec{\sigma}-\vec{\sigma}')\end{aligned} \tag{62}$$

and for the generalized conformal boost generator densities $\widetilde{K}$

$$\begin{aligned}&\{\widetilde{\Phi}^{\alpha\beta}(\vec{\sigma}),\widetilde{K}_{\gamma\delta}(\vec{\sigma}')\}_{P.B.}=\tfrac{1}{2}\Big(\delta^{\alpha}_{\gamma}[-2(z_{\delta}{}^{\lambda}+2i\theta_{\delta}\theta^{\lambda})\widetilde{\Phi}_{\lambda}{}^{\beta}+(x_{\delta\dot{\lambda}}+2i\theta_{\delta}\bar{\theta}_{\dot{\lambda}})\widetilde{\Phi}^{\dot{\lambda}\beta}+\theta_{\delta}\widetilde{\Psi}^{\beta}] \\ &+\delta^{\alpha}_{\delta}[-2(z_{\gamma}{}^{\lambda}+2i\theta_{\gamma}\theta^{\lambda})\widetilde{\Phi}_{\lambda}{}^{\beta}+(x_{\gamma\dot{\lambda}}+2i\theta_{\gamma}\bar{\theta}_{\dot{\lambda}})\widetilde{\Phi}^{\dot{\lambda}\beta}+\theta_{\gamma}\widetilde{\Psi}^{\beta}]+(\alpha\leftrightarrow\beta)\Big)\,\delta^{p}(\vec{\sigma}-\vec{\sigma}'), \\ &\{\widetilde{\Phi}^{\alpha\beta}(\vec{\sigma}),K_{\gamma\dot{\gamma}}(\vec{\sigma}')\}_{P.B.}=\tfrac{1}{2}\Big(\delta^{\alpha}_{\gamma}[2(x_{\lambda\dot{\gamma}}+2i\bar{\theta}_{\dot{\gamma}}\theta_{\lambda})\widetilde{\Phi}^{\beta\lambda}-(\bar{z}_{\dot{\gamma}}{}^{\dot{\lambda}}+2i\bar{\theta}_{\dot{\gamma}}\bar{\theta}^{\dot{\lambda}})\widetilde{\Phi}^{\beta}{}_{\dot{\lambda}}+\bar{\theta}_{\dot{\gamma}}\widetilde{\Psi}^{\beta}] \\ &\qquad+(\alpha\leftrightarrow\beta)\big)\,\delta^{p}(\vec{\sigma}-\vec{\sigma}'); \\ &\{\widetilde{\Phi}^{\dot{\alpha}\alpha}(\vec{\sigma}),K_{\gamma\delta}(\vec{\sigma}')\}_{P.B.}=\Big(\delta^{\alpha}_{\gamma}[-(z_{\delta}{}^{\lambda}+2i\theta_{\delta}\theta^{\lambda})\widetilde{\Phi}_{\lambda}{}^{\dot{\alpha}}+2(x_{\delta\dot{\lambda}}+2i\theta_{\delta}\bar{\theta}_{\dot{\lambda}})\bar{\widetilde{\Phi}}^{\dot{\lambda}\dot{\alpha}}+\theta_{\delta}\bar{\widetilde{\Psi}}^{\dot{\alpha}}] \\ &\qquad+(\gamma\leftrightarrow\delta)\big)\,\delta^{p}(\vec{\sigma}-\vec{\sigma}'), \\ &\{\widetilde{\Phi}^{\dot{\alpha}\alpha}(\vec{\sigma}),K_{\gamma\dot{\gamma}}(\vec{\sigma}')\}_{P.B.}=\Big(\delta^{\alpha}_{\gamma}[(x_{\lambda\dot{\gamma}}+2i\bar{\theta}_{\dot{\gamma}}\theta_{\lambda})\widetilde{\Phi}^{\dot{\alpha}\lambda}+2(\bar{z}_{\dot{\gamma}\dot{\lambda}}+2i\bar{\theta}_{\dot{\gamma}}\bar{\theta}_{\dot{\lambda}})\bar{\widetilde{\Phi}}^{\dot{\lambda}\dot{\alpha}}+\bar{\theta}_{\dot{\gamma}}\bar{\widetilde{\Psi}}^{\dot{\alpha}}] \\ &\quad+\delta^{\dot{\alpha}}_{\dot{\gamma}}[-2(z_{\gamma}{}^{\lambda}+2i\theta_{\gamma}\theta^{\lambda})\widetilde{\Phi}_{\lambda}{}^{\alpha}+(x_{\gamma\dot{\lambda}}+2i\theta_{\gamma}\bar{\theta}_{\dot{\lambda}})\widetilde{\Phi}^{\dot{\lambda}\alpha}+\theta_{\gamma}\widetilde{\Psi}^{\alpha}]\Big)\,\delta^{p}(\vec{\sigma}-\vec{\sigma}').\end{aligned} \tag{63}$$

Secondly, the $\widetilde{\Psi}$-constraints have the following nonzero P.B's. with the $OSp(1|8)$ generator densities

$$\begin{gathered}\{\widetilde{\Psi}_{\alpha}(\vec{\sigma}),\widetilde{L}_{\beta\gamma}(\vec{\sigma}')\}_{P.B.}=\varepsilon_{\alpha\gamma}\widetilde{\Psi}_{\beta}\delta^{p}(\vec{\sigma}-\vec{\sigma}'),\ \{\widetilde{\Psi}_{\alpha}(\vec{\sigma}),\widetilde{L}_{\dot{\beta}\gamma}(\vec{\sigma}')\}_{P.B.}=\varepsilon_{\alpha\gamma}\bar{\widetilde{\Psi}}_{\dot{\beta}}\delta^{p}(\vec{\sigma}-\vec{\sigma}'); \\ \{\widetilde{\Psi}_{\alpha}(\vec{\sigma}),\widetilde{S}_{\gamma}(\vec{\sigma}')\}_{P.B.}=4i\varepsilon_{\alpha\gamma}(\widetilde{\Psi}^{\beta}\theta_{\beta}+\bar{\widetilde{\Psi}}^{\dot{\beta}}\bar{\theta}_{\dot{\beta}})\delta^{p}(\vec{\sigma}-\vec{\sigma}'); \\ \{\widetilde{\Psi}_{\alpha}(\vec{\sigma}),\widetilde{K}_{\gamma\delta}(\vec{\sigma}')\}_{P.B.}=\Big(\varepsilon_{\alpha\delta}[(z_{\gamma\beta}+2i\theta_{\gamma}\theta_{\beta})\widetilde{\Psi}^{\beta}+(x_{\gamma\dot{\beta}}+2i\theta_{\gamma}\bar{\theta}_{\dot{\beta}})\bar{\widetilde{\Psi}}^{\dot{\beta}}]+(\gamma\leftrightarrow\delta)\Big)\,\delta^{p}(\vec{\sigma}-\vec{\sigma}'), \\ \{\widetilde{\Psi}_{\alpha}(\vec{\sigma}),\widetilde{K}_{\gamma\dot{\gamma}}(\vec{\sigma}')\}_{P.B.}=\varepsilon_{\alpha\gamma}[(x_{\beta\dot{\gamma}}+2i\bar{\theta}_{\dot{\gamma}}\theta_{\beta})\widetilde{\Psi}^{\beta}+(\bar{z}_{\dot{\gamma}\dot{\beta}}+2i\bar{\theta}_{\dot{\gamma}}\bar{\theta}_{\dot{\beta}})\bar{\widetilde{\Psi}}^{\dot{\beta}}]\delta^{p}(\vec{\sigma}-\vec{\sigma}').\end{gathered} \tag{64}$$

The above expressions should be complemented by their complex conjugate.

The Poisson brackets of the $OSp(1|8)$ generator densities denoted collectively by $\widetilde{G}=(Q_{\alpha},\bar{Q}_{\dot{\alpha}},\widetilde{S}_{\alpha},\bar{\widetilde{S}}_{\dot{\alpha}},\pi_{\alpha\beta},\bar{\pi}_{\dot{\alpha}\dot{\beta}},P_{\alpha\dot{\beta}},\widetilde{L}_{\alpha\beta},\bar{\widetilde{L}}_{\dot{\alpha}\dot{\beta}},\widetilde{L}_{\alpha\dot{\beta}},\widetilde{L}_{\dot{\beta}\alpha},\widetilde{K}_{\alpha\beta},\bar{\widetilde{K}}_{\dot{\alpha}\dot{\beta}},\widetilde{K}_{\alpha\dot{\beta}})$ with the $\vec{\sigma}$-reparametrization generators $\widetilde{L}_{M}$ are equal to

$$\{\widetilde{L}_{M}(\vec{\sigma}),G(\vec{\sigma}')\}_{P.B.}=-G(\vec{\sigma})\partial_{M}\delta^{p}(\vec{\sigma}-\vec{\sigma}'). \tag{65}$$

The $OSp(1|8)$ generator densities strongly commute with the converted first-class constraints $\widetilde{\Delta}_{W}$, $\widetilde{P}_{u\alpha}$, $\bar{\widetilde{P}}_{u\dot{\alpha}}$, $\widetilde{P}^{(\rho)}_{\tau}$ and $P^{(\rho)}_{M}$.

Thus, we conclude that the $OSp(1|8)$ generators are gauge invariant quantities on the classical level and the next step is to study this gauge invariance on the quantum level.

6 Quantum superalgebra of global and gauge symmetries in the extended phase space. Conformal anomaly?

To lift the considerations of the previous Section to the quantum level one has to consider all the quantities entering expressions for the first-class constraints (29)-(32), (36), (38) and $OSp(1|8)$ generator densities (47), (50), (52), (53), (55) as operators and to choose certain ordering prescription for them. The next step of the quantum theory consistency check is to verify the validity of $OSp(1|8)$ superalgebra (anti)commutation relations and the gauge invariant character of generator densities. For definiteness let start below considering $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering. So that constraints and generator densities are the sums of monomials of the form $\Pi\hat{\mathcal{Q}}\Pi\hat{\mathcal{P}} = \hat{\mathcal{Q}}_1^{n_1}\hat{\mathcal{Q}}_2^{n_2}\ldots\hat{\mathcal{Q}}_k^{n_k}\hat{\mathcal{P}}_1^{m_1}\hat{\mathcal{P}}_2^{m_2}\ldots\hat{\mathcal{P}}_l^{m_l}$ and the chosen ordering will be preserved in course of calculation of (anti)commutators if it is preserved in "elementary" (anti)commutators $[\Pi\hat{\mathcal{Q}},\Pi\hat{\mathcal{P}}\}$, $[\Pi\hat{\mathcal{P}},\Pi\hat{\mathcal{Q}}\}$. It was argued in [43] that the sufficient condition for ordering preservation is the absence for any $\hat{\mathcal{Q}}$-monomial of the corresponding $\hat{\mathcal{P}}$-monomial(s) containing more than one momentum variable conjugate to that of $\hat{\mathcal{Q}}$-monomial. As is readily seen from the expressions for the constraints (29)-(32), (36), (38) and $OSp(1|8)$ generator densities (47), (50), (52), (53), (55) there are present only $\hat{\mathcal{P}}$ monomials of the first power satisfying automatically the above criterion except for $\hat{\tilde{P}}_{u\gamma}\hat{\tilde{P}}_{u\lambda}$, $\hat{\tilde{P}}_{u\gamma}\hat{\bar{\tilde{P}}}_{u\dot{\gamma}}$ monomials entering the generalized conformal transformations generator densities $\hat{\widetilde{K}}_{\gamma\lambda}(\tau,\vec{\sigma})$, $\hat{\widetilde{K}}_{\gamma\dot{\gamma}}(\tau,\vec{\sigma})$ (52), (53). Their conjugate $\hat{\mathcal{Q}}$ monomials $\hat{\tilde{u}}_\alpha\hat{\tilde{u}}_\beta$, $\hat{\tilde{u}}_\alpha\hat{\bar{\tilde{u}}}_{\dot{\beta}}$ enter $\hat{\tilde{\Phi}}$-constraints (31) and direct calculation reveals anomalous contributions (with $\hbar = c = 1$) to the classical expressions (63)

$$
\begin{aligned}
[\hat{\tilde{\Phi}}^{\alpha\beta}(\vec{\sigma}),\hat{\tilde{K}}_{\gamma\lambda}(\vec{\sigma}')]|_{\text{anomal. contr.}} &= -\tfrac{1}{2}(\delta^\alpha_\gamma\delta^\beta_\lambda+\delta^\alpha_\lambda\delta^\beta_\gamma)\delta^p_\epsilon(\vec{\sigma}-\vec{\sigma}')\delta^p_\epsilon(0),\\
[\hat{\tilde{\Phi}}^{\dot{\alpha}\alpha}(\vec{\sigma}),\hat{\tilde{K}}_{\gamma\dot{\gamma}}(\vec{\sigma}')]|_{\text{anomal. contr.}} &= -\delta^\alpha_\gamma\delta^{\dot{\alpha}}_{\dot{\gamma}}\delta^p_\epsilon(\vec{\sigma}-\vec{\sigma}')\delta^p_\epsilon(0)
\end{aligned}
\tag{66}
$$

and their c. c. relations, where the P.B's. were changed by the commutators

$$
[\hat{\tilde{P}}_{u\alpha}(\vec{\sigma}),\hat{\tilde{u}}^\beta(\vec{\sigma}')] = i\delta^\beta_\alpha\delta^p_\epsilon(\vec{\sigma}-\vec{\sigma}'),\ [\hat{\bar{\tilde{P}}}_{u\dot{\alpha}}(\vec{\sigma}),\hat{\bar{\tilde{u}}}^{\dot{\beta}}(\vec{\sigma}')] = i\delta^{\dot{\beta}}_{\dot{\alpha}}\delta^p_\epsilon(\vec{\sigma}-\vec{\sigma}')
\tag{67}
$$

including the regularized delta function $\delta^p_\epsilon(\vec{\sigma}-\vec{\sigma}')$ considered in [48]. This means that the quantum conformal boosts are not gauge invariant operators and the generalized conformal symmetry is broken. However, this breaking could be a consequence of the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering prescription used for the transition to quantum theory[1]. In fact, the anomalous terms may be hidden if we consider the hermitian expressions for the generator densities. These expressions differ from the above considered $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered expressions (47),(50), (52), (53), (55) by the addition of singular terms proportional to $\delta^p_\epsilon(0)$

$$
\begin{gathered}
\hat{\tilde{\mathcal{S}}}_\gamma = \hat{\tilde{S}}_\gamma + 6\theta_\gamma\delta^p_\epsilon(0),\ \hat{\bar{\tilde{\mathcal{S}}}}_{\dot{\gamma}} = \hat{\bar{\tilde{S}}}_{\dot{\gamma}} + 6\bar{\theta}_{\dot{\gamma}}\delta^p_\epsilon(0),\\
\hat{\tilde{\mathcal{K}}}_{\gamma\lambda} = \hat{\tilde{K}}_{\gamma\lambda} - 3iz_{\gamma\lambda}\delta^p_\epsilon(0),\ \hat{\bar{\tilde{\mathcal{K}}}}_{\dot{\gamma}\dot{\lambda}} = \hat{\bar{\tilde{K}}}_{\dot{\gamma}\dot{\lambda}} - 3i\bar{z}_{\dot{\gamma}\dot{\lambda}}\delta^p_\epsilon(0),\ \hat{\tilde{\mathcal{K}}}_{\gamma\dot{\gamma}} = \hat{\tilde{K}}_{\gamma\dot{\gamma}} - 3ix_{\gamma\dot{\gamma}}\delta^p_\epsilon(0),\\
\hat{\tilde{\mathcal{L}}}_{\alpha\beta} = \hat{\tilde{L}}_{\alpha\beta} - \tfrac{3i}{2}\varepsilon_{\alpha\beta}\delta^p_\epsilon(0),\ \hat{\bar{\tilde{\mathcal{L}}}}_{\dot{\alpha}\dot{\beta}} = \hat{\bar{\tilde{L}}}_{\dot{\alpha}\dot{\beta}} - \tfrac{3i}{2}\bar{\varepsilon}_{\dot{\alpha}\dot{\beta}}\delta^p_\epsilon(0).
\end{gathered}
\tag{68}
$$

[1] It was noted by D. Sorokin with the reference on the paper [28], where a modified realization of the conformal generators was considered for the quantum superparticle without breaking of the conformal supersymmetry.

These singular terms appear as a result of the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering in the non-ordered hermitian expressions for the generators and using the relations similar to

$$\frac{1}{2}(z_{\lambda\delta}(\vec{\sigma})\pi^{\delta\varepsilon}(\vec{\sigma})+\pi^{\delta\varepsilon}(\vec{\sigma})z_{\lambda\delta}(\vec{\sigma}))=z_{\lambda\delta}(\vec{\sigma})\pi^{\delta\varepsilon}(\vec{\sigma})-\frac{3i}{4}\delta^{\varepsilon}_{\lambda}\delta^{p}_{\epsilon}(0) \tag{69}$$

for all products of the brane coordinates and momenta. Then we find that the substitution of $\hat{\tilde{\mathcal{K}}}$ (68) for $\hat{\tilde{K}}$ in the commutators (66) will change only the coefficients there

$$\begin{aligned}
[\hat{\tilde{\Phi}}^{\alpha\beta}(\vec{\sigma}),\hat{\tilde{\mathcal{K}}}_{\gamma\lambda}(\vec{\sigma}')]|_{\text{anomal. contr.}}&=-(\tfrac{1}{2}+\tfrac{3}{2})(\delta^{\alpha}_{\gamma}\delta^{\beta}_{\lambda}+\delta^{\alpha}_{\lambda}\delta^{\beta}_{\gamma})\delta^{p}_{\epsilon}(\vec{\sigma}-\vec{\sigma}')\delta^{p}_{\epsilon}(0),\\
[\hat{\tilde{\Phi}}^{\dot{\alpha}\alpha}(\vec{\sigma}),\hat{\tilde{\mathcal{K}}}_{\gamma\dot{\gamma}}(\vec{\sigma}')]|_{\text{anomal. contr.}}&=-(1+3)\delta^{\alpha}_{\gamma}\delta^{\dot{\alpha}}_{\dot{\gamma}}\delta^{p}_{\epsilon}(\vec{\sigma}-\vec{\sigma}')\delta^{p}_{\epsilon}(0)
\end{aligned} \tag{70}$$

and in their c. c. commutators. To derive this result the changes $\delta^{p}(\vec{\sigma}-\vec{\sigma}')\rightarrow\delta^{p}_{\epsilon}(\vec{\sigma}-\vec{\sigma}')$ and $\mathrm{i}\{\mathcal{P},\mathcal{Q}\}_{\mathcal{P.B.}}\rightarrow[\ \hat{\mathcal{P}},\hat{\mathcal{Q}}\ ]$ in the canonical P.B's. (12) were taken into account.

Then we revealed that the singular terms in the r.h.s. of (70) are precisely the terms which are needed for the hermiticity restoration of the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered non-anomalous operators in the r.h.s. (63). We found that the same effect is valid for other (anti)commutators of the algebra. Thus, we have shown that the considered singular terms in (66) and (70) may be hidden in the expressions for the disordered hermitian generators. If we have started from this hermitian representation we could not observe the singular terms.

So, it seems that namely these non-ordered definitions might be accepted as correct quantum definitions of the conformal hermitian generators. However, it is not end of the story, because of the necessity to define the relevant vacuum state and to construct both the proper physical space of states associated with the considered BPS branes and nilpotent BRST operator. This problem is open for the string or brane in contrast to the superparticle [28], because there the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordered hermitian superconformal generators contain only finite central terms instead of the singular terms under question. It is clear, because p-brane is a field object with infinite number of degrees of freedom. Similar problem was studied in the series of papers [48],[49] devoted to the light-cone quantization of tensionless string. The starting point there was consideration of hermitian expressions for the generators and constraints which have further been transformed to the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-order. This ordering was chosen as physical one consistent with the definition of the vacuum state as a state annihilated by the momentum operator. The definition [49] of the vacuum state was physically justified by the analysis of the tensionless limit of the vacuum conditions accepted in the tensile string theory. As a result, the conformal symmetry breaking was revealed there for the space-time dimension $D>2$.

7 Conclusion

In the present report we studied the classical and quantum symmetries of the recently proposed tensionless super p-brane model [31] in $D=4$ $N=1$ superspace extended by TCC coordinates z_{mn}. The primary and secondary constraints of the model form a mixed set of the first- and second-class constraints and the Dirac bracket realization of the global and gauge symmetries results in rather nonlinear expressions obtained in [39]. One of the possible methods to get rid of the second-class constraints, and respectively to avoid the nonlinearities, is their conversion to the first-class constraints using additional conversion variables [40]-[42] and preserving the number of the primary physical degrees of freedom.

We applied this method here to the primary and secondary constraints of the model [31] and transformed them into a set of the converted first-class constraints. As a result, we found the classical realization of the $OSp(1|8)$ generators in extended phase space and established their invariance under the world-volume symmetry transformations.

For the transition to quantum theory we studied the problem of ordering of the generalized $\hat{\mathcal{Q}}$ and $\hat{\mathcal{P}}$ operators of the brane. We found that the hermitian expressions for $OSp(1|8)$ generators without $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering of the operators forming them may be considered as a quantum generalization of the corresponding classical generators. Together with the generators of the gauge world-volume symmetry they form closed quantum algebra of local and global brane symmetries free of anomalous terms.

However, the problem appears how to construct the relevant vacuum state and proper physical space for the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-desordered quantum generators and constraints. Transition to the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering in the hermitian expressions for the $OSp(1|8)$ quantum generators is probably able to weaken the mentioned problem, but then we need to give a proper physical interpretation of the above mentioned singular terms in the generators and commutators. It is possible that the analyzed quantum deviations could imply explicit breakdown of the classical superconformal symmetry $OSp(1|8)$ after quantization if the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering might be approved as physical one. That possibility has been realized in [49] for tensionless strings, having no oscillator degrees of freedom, where the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering was justified to be physical. It was shown there that this ordering is consistent with the existence of nilpotent BRST operator and the absence of divergent anomalous terms only in the space-time with dimension $D = 2$. Note that, as it follows from (68), the contributions of the spinor, vector and tensor degrees of freedom to the brane anomaly were obtained namely for the $D = 4$ case which is in the black list of [49]. Let us also remind that the conformal invariance for massless particle in $D > 2$ survives the transition to the quantum case, as it was also subscribed in [49]. This conclusion agrees with the above mentioned result of [28], where superconformal generators of superparticle include only finite terms resulting from the $\hat{\mathcal{Q}}\hat{\mathcal{P}}$-ordering. For the brane case, however the similar terms are divergent, as it follows from (68) and (69) if the regularization in the delta functions is removed, i.e. the limit $\epsilon \to 0$ is implemented. Thus, to prove that $OSp(1|8)$ conformal supersymmetry survives the quantization with the considered disordered hermitian $OSp(1|8)$ generators and world-volume gauge constraints one has to construct proper vacuum state and physical space of states permitting nilpotent BRST charge.

As a result, the question whether the global $OSp(1|8)$ supersymmetry is proper quantum symmetry of the exotic BPS branes needs in further investigation. Finally note that it is very interesting to apply our analysis to exotic $OSp(1|64)$ invariant superbranes of M-theory in $N = 1$ $D = 11$ superspace enlarged by TCC coordinates.

8 Acknowledgements

The authors are grateful to the Organizers of SQS Workshop for the warm hospitality in Dubna and I. Bengtsson, A. Laudal, M. Movshev, A. Pashnev, D. Sorokin and B. Sundborg for useful remarks and fruitful discussions. A.Z. thanks Fysikum at the Stockholm University and the Mittag-Leffler Institute for kind hospitality. The work was partially supported by the grant of the Royal Swedish Academy of Sciences and by the SFFR of Ukraine under Project 02.07.276.

References

[1] M.J. Duff and K. Stelle, *Multimembrane solution of D=11 supegravity*, Phys. Lett. **B253** (1991) 113;
M.J. Duff, *M-theory on manifolds of G_2 holonomy: the first twenty years*, hep-th/0201062.

[2] M.J. Duff and J.M. Liu, *Hidden Spacetime Symmetries and Generalized Holonomy in M-theory*, Nucl. Phys. **B674** (2003) 217, hep-th/0303140.

[3] C.M. Hull, *Holonomy and Symmetry in M-theory*, hep-th/0305039.

[4] R. d'Auria and P. Fre, *Geometric supergravity in D=11 and its hidden supergroup*, Nucl. Phys. **B201** (1982) 101, erratum *ibid* **206** (1982) 496.

[5] J. van Holten and A. van Proeyen, *N=1 supersymmetry algebras in D=2,3,4(mod)8*, J. Phys. **A15** (1982) 3763.

[6] P.A. Zizzi, *Antisymmetric tensors in supersymmetry algebras and spontaneous compactification*, Phys. Lett. **B149** (1984) 333.

[7] J. Hughes and J. Polchinski, *Partially broken global supersymmetry and the superstring*, Nucl. Phys. **B278** (1986) 147.

[8] J. Hughes, J. Liu and J. Polchinski, *Supermembranes*, Phys. Lett. **B180** (1986) 370.

[9] T. Curtright, *Are there any superstrings in eleven-dimensions?* Phys. Rev. Lett. **60** (1988) 393.

[10] J.A. de Azcarraga, J.P. Gauntlett, J.M. Izquierdo and P.K. Townsend, *Topological extensions of the supersymmetry algebra for extended objects*, Phys. Rev. Lett. **63** (1989) 2443.

[11] Y. Eisenberg and S. Solomon, *(Super)field theories from (super)twistors*, Phys. Lett. **B220** (1989) 113.

[12] M.B. Green, *Supertranslations, superstrings and Chern-Simons forms*, Phys. Lett. **B223** (1989) 157.

[13] W. Siegel, *Randomizing the superstring*, Phys. Rev. **D50** (1994) 2799; hep-th/9403144.

[14] E. Bergshoeff and E. Sezgin, *Super$-p-$brane theories and new space-time superalgebras*, Phys. Lett. **B354** (1995) 256; hep-th/9504140.

[15] H. Lu and C.N. Pope, *Multiscalar $p-$brane solitons*, Int. J. Mod. Phys. **A12** (1997) 437; hep-th/9512153.

[16] E. Sezgin, *The M-algebra*, Phys. Lett. **B392** (1997) 323; hep-th/9609086.

[17] I. Rudychev and E. Sezgin, *Superparticles in $D > 11$*, Phys. Lett. **B415** (1997) 363; hep-th/9704057; addendum Phys. Lett. **B424** (1998) 411; hep-th/9711128.

[18] C. Chryssomalakos, J.A. de Azcarraga, J.M. Izquierdo, J.C. Perez Bueno, *The geometry of branes and extended superspaces*, Nucl. Phys. **B567** (2000) 293; hep-th/9904137.

[19] I. Bandos and J. Lukierski, *Tensorial central charges and new superparticle models with fundamental spinor coordinates*, Mod. Phys. Lett. **A14** (1999) 1257, hep-th/9811022.

[20] T. Ueno, *BPS states in* $10+2$ *dimensions*, JHEP **0012** (2000) 006; hep-th/9909007.

[21] J.P. Gauntlett and C.M. Hull, *BPS states with extra supersymmetry*, JHEP **0001** (2000) 004; hep-th/9909098.

[22] J.P. Gauntlett, G. Gibbons, C.M. Hull and P.K. Townsend, *BPS states of D=4 N=1 supersymmetry*, Comm. Math. Phys. **216** (2001) 431; hep-th/0001024.

[23] M. Blau, J. Figueroa-O'Farrill, C. Hull, and G. Papadopoulos, *A new maximally supersymmetric background of IIB superstring theory*, JHEP **0201** (2002) 047; hep-th/0110242;
Penrose limits and maximal supersymmetry, Class. Quant. Grav. **19** (2002) L87; hep-th/0201081.

[24] M. Cvetič, H. Lü and C.N. Pope, *Penrose limits, pp-waves and deformed M2-branes*, hep-th/0203082;
M-theory pp-waves, Penrose limits and supernumerary supersymmetries, Nucl. Phys. **B644** (2002) 65; hep-th/0203229.

[25] J. Michelson, *Twisted toroidal compactification of pp-waves*, Phys. Rev. **D66** (2002) 06002; hep-th/0203140.

[26] J.P. Gauntlett and C.M. Hull, *pp-Waves in 11-dimensions with extra supersymmetry*, JHEP **0206** (2002) 013; hep-th/0203255.

[27] C. Fronsdal, Massless particles, orthosymplectic symmetry and another type of Kaluza-Klein theory, in "Essays on supersymmetry", Editor C. Fronsdal, Dordrecht, Reidel, 1986, Math. Phys. Studies, V.8, 164.

[28] M.A. Vasiliev, *Conformal higher spin symmetries of 4d massless supermultiplets and* $osp(L,2M)$ *invariant equations in generalized (super)space*, Phys. Rev. **D66** (2002) 066006; hep-th/0106149.

[29] B. Sundborg, *Stringy gravity, interacting tensionless strings and massless higher spins*, Nucl. Phys. B (Proc. Suppl.) **102-103** (2001) 113, hep-th/0103247.

[30] E. Witten, *Spacetime reconstruction*, unpublished,
see http://theory.caltech.edu/jhs60/witten/1.html

[31] A.A. Zheltukhin and D.V. Uvarov, *An inverse Penrose limit and supersymmetry enhancement in the presence of tensor central charges*, JHEP **08** (2002) 008; *Exactly solvable super-p-brane models with tensor central charge coordinates*, Phys. Lett. **B545** (2002) 183, hep-th/0206214.

[32] A.A. Zheltukhin and D.V. Uvarov, *Extra gauge symmetry for extra supersymmetry*, Phys. Lett. **B565** (2003) 229; hep-th/0304151.

[33] A. Ferber, *Supertwistors and conformal supersymmetry*, Nucl. Phys. **B132** (1978) 55.

[34] T. Shirafuji, *Lagrangian mechanics of massless particles with spin*, Prog. Theor. Phys. **70** (1983) 18.

[35] I. Bandos, *BPS preons and tensionless super-p-brane in generalized superspace*, Phys. Lett. **B558** (2003) 197, hep-th/0208110.

[36] I.A. Bandos, J.A. de Azcarrage, M. Picon and O. Varela, *D=11 superstring model with 30 κ-symmetries and $\frac{30}{32}$ BPS states in an extended superspace*, hep-th/0307106.

[37] I. Bengtsson and A.A. Zheltukhin, *Wess-Zumino actions and Dirichlet boundary conditions for super p-branes with exotic fractions of supersymmetry*, Phys. Lett. **B570** (2003) 222; hep-th/0306172.

[38] P.C. West, *Hidden superconformal symmetry in M theory*, JHEP **08** (2000) 007, hep-th/0005270.

[39] D.V. Uvarov and A.A. Zheltukhin, *Hamiltonian of superstring and super p-branes with enhanced supersymmetry*, to appear in the Proceedings of the $V^{\rm th}$ International Conference "Symmetries in Nonlinear Mathematical Physics", June 24-29, 2003, Kyiv, Ukraine;
Hamiltonian structure and noncommutativity in p-brane models with exotic supersymmetry, hep-th/0310284.

[40] L.D. Faddeev and S.L. Shatashvili, *Realization of the Schwinger term in the Gauss law and the possibility of correct quantization of a theory with anomalies*, Phys. Lett. **B167** (1986) 225.

[41] I.A. Batalin and E.S. Fradkin, *Operatorial quantization of dynamical systems subject to second class constraints*, Nucl. Phys. **B279** (1987) 538;
I.A. Batalin, E.S. Fradkin and T.E. Fradkina, *Another version for operatorial quantization of dynamical systems with irreducible constraints*, Nucl. Phys. **B314** (1989) 158.

[42] E.Sh. Egorian and R.P. Manvelian, *Quantization of dynamical systems with second class constraints*, Theor. Math. Phys. **94** (1993) 241.

[43] I.A. Bandos and A.A. Zheltukhin, *Hamiltonian mechanics and absence of critical dimensions for null membranes* Sov. J. Nucl. Phys. **50** (1989) 556;
I.A. Bandos and A.A. Zheltukhin, *Null super p-branes quantum theory in 4-dimensional space-time*, Fortschr. Phys. **41** (1993) 619.

[44] Y. Eisenberg and S. Solomon, *The twistor geometry of the covariantly quantized Brink-Schwarz superparticle* Nucl. Phys. **B309** (1988) 709.

[45] M.S. Plyushchay, *Covariant quantization of massles superparticle in 4-dimensional space-time: twistor approach* Mod. Phys. Lett. **A4** (1989) 1827.

[46] I. Bandos, A. Maznytsia, I. Rudychev and D. Sorokin, *On the BRST quantization of the massless bosonic particle in twistor-like approach* Int. J. Mod. Phys. **A12** (1997) 3259, hep-th/9609107.

[47] I.A. Bandos, J. Lukierski and D.P. Sorokin, *Superparticle models with tensorial central charges*, Phys. Rev. **D61**(2000) 045002; hep-th/9904109.

[48] J. Isberg, U. Lindström and B. Sundborg, *Space-time symmetries of quantized tensionless strings*, Phys. Lett. **293** (1992) 321, hep-th/9207005;
J. Isberg, U. Lindström, B. Sundborg and G. Theodoridis, *Classical and quantized tensionless strings*, Nucl. Phys. **B411** (1994) 122, hep-th/9307108.

[49] H. Gustafsson, U. Lindström, P Salsidis, B. Sundborg and R. von Unge, *Hamiltonian BRST quantization of the conformal string*, Nucl. Phys. **B440** (1995) 495.